# MEM6810 Engineering Systems Modeling and Simulation

Sino-US Global Logistics Institute Shanghai Jiao Tong University

Spring 2025 (full-time)

# Assignment 5

Due Date: June 27 (14:00)

#### Instruction

- (a) You can answer in English or Chinese or both.
- (b) Show **enough** intermediate steps.
- (c) Write by hand.
- (d) Upload the answers in the Canvas system; late submission will not be accepted.

## Question 1 (40 points)

We have k > 2 different (system) designs, and their mean performances are  $\theta_i$ , i = 1, 2, ..., k. We want to select the one with the largest mean performance. Bechhofer's Procedure (Lec 9 page 19/29) can ensure that when Assumptions 1-4 (Lec 9 page 18/29) are satisfied,  $\mathbb{P}$ {select the larget  $\theta_i$ }  $\geq 1 - \alpha$ . Now we relax Assumption 3. Give a rigorous proof that, when Assumptions 1, 2, and 4 (Lec 9 page 18/29) are satisfied,

$$\mathbb{P}\left\{\left|\text{selected } \theta_i - \max_{1 \le i \le k} \theta_i\right| < \delta\right\} \ge 1 - \alpha.$$

### Question 2 (60 points)

Consider the simulation optimization problem,

$$\min_{\boldsymbol{x}\in\mathcal{X}} g(\boldsymbol{x}),$$

where  $g(\boldsymbol{x}) \coloneqq \mathbb{E}[G(\boldsymbol{x})]$  and  $G(\boldsymbol{x})$  is the output of a simulation replication conducted at  $\boldsymbol{x}$ . Let  $\boldsymbol{x}^*$  be a global optimal solution. Grid search is often used to find a global optimal solution to the problem. It first chooses m grid points,  $\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_m$ , in  $\mathcal{X}$ . It then takes r i.i.d. observations from each of the m grid points and calculates the sample means,  $\bar{G}(\boldsymbol{x}_1), \bar{G}(\boldsymbol{x}_2), \cdots, \bar{G}(\boldsymbol{x}_m)$ . Let

$$\hat{\boldsymbol{x}}_m^* = \arg\min\{\bar{G}(\boldsymbol{x}_1), \bar{G}(\boldsymbol{x}_2), \cdots, \bar{G}(\boldsymbol{x}_m)\} \text{ and } \boldsymbol{x}_m^* = \arg\min\{g(\boldsymbol{x}_1), g(\boldsymbol{x}_2), \cdots, g(\boldsymbol{x}_m)\}$$

Suppose that the grid points are chosen such that  $g(\boldsymbol{x}_m^*) \to g(\boldsymbol{x}^*)$  as  $m \to \infty$ . (How to ensure the above condition is of course an important question in practice. Here we

simply assume we can do it.) We further assume that  $\sup_{\boldsymbol{x}\in\mathcal{X}} \operatorname{Var}[G(\boldsymbol{x})] = \sigma^2 < \infty$ . In order to ensure that  $g(\hat{\boldsymbol{x}}_m^*) \to g(\boldsymbol{x}^*)$  almost surely as  $m \to \infty$ , r and m need to satisfy some relationship. Prove that, if r will increase when m increases (that is to say r = r(m) is an increasing function on m) and

$$\sum_{m=1}^{\infty} \frac{m}{r(m)} < \infty,$$

then  $g(\hat{\boldsymbol{x}}_m^*) \to g(\boldsymbol{x}^*)$  almost surely as  $m \to \infty$ .

**Hint**: You may need to use the following fact:  $|\min_{i=1,...,k} \{a_i\} - \min_{i=1,...,k} \{b_i\}| \le \max_{i=1,...,k} \{|a_i-b_i|\}$ , for any given  $\{a_1,...,a_k\}$  and  $\{b_1,...,b_k\}$ . You can find some idea from [L. Jeff Hong, Barry L. Nelson (2006). Discrete optimization via simulation using COMPASS. *Operations Research* **54**(1):115-129. https://doi.org/10.1287/opre.1050.0237]